Collective effects in the energy loss of ion beams in fusion plasmas

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A model to describe the collective effects in the energy loss of intense ion beams or large ion clusters in plasmas is formulated. Important interference effects are obtained, which are produced by the dynamical vicinage interactions between the beam particles. They are represented in an average statistical way using a pair-correlation function to describe a bunch of particles in the beam, and using the dielectric function approach to represent collective and individual excitations in the plasma. The magnitude and the characteristics of the collective effect in the energy loss are illustrated for several cases of beam-plasma interactions in the range of current experiments using high-intensity ion beams. A strong enhancement in the energy loss values is obtained for intermediate beam or cluster sizes.

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The use of intense particle beams has become an important method for the heating of plasmas in fusion research. High-intensity ion beams are used in inertial-confinement fusion studies (ICF), where they provide favorable conditions of high-energy concentrations and fast repetition rates [1–3]. In a different situation, neutral atomic beams are currently used in Tokamak experiments as an efficient method to bring the plasma temperature into the range required for D-T fusion tests [4,5].

Most of the experimental and theoretical studies on beam-plasma interactions consider beams of atomic particles where the separation among each of these particles is so large that the medium goes back to equilibrium before a new particle enters a given interaction region. This situation can be represented by the condition $d \gg \lambda$, where d is an average distance between neighboring particles in the beam and λ is of the order of the dynamical screening distance in the plasma (i.e., the effective interaction range).

Studies of collective effects in the interaction between swift ion clusters and matter have been made for both solids [6–9] and plasma targets [10,11]. The possibility of producing high-intensity beams of light or heavy ions has stimulated great interest in feasibility studies on the use of intense beams to approach fusion conditions.

According to recent estimates [12–14], consideration of collective effects may be relevant to account for the energy deposition by available high-intensity ion beams, or by cluster impact on dense plasmas.

The purpose of this paper is to provide a quantitative basis to evaluate the collective effects in the energy loss of intense ion beams in fusion plasmas. We concentrate here on the description of collective effects for very large clusters, corresponding to the distribution of particles within a given bunch of a high-intensity ion beam. We will show how the correlation between the particles in the beam has an important influence on the interference terms that give rise to the collective effects in the beam-plasma interaction. The magnitude of these effects will be illustrated with calculations for some cases of interest.

The interaction between a cluster of ions and a dispersive medium can be studied in a closed form using the dielectric-function formalism [6–9]. The mean energy loss for a beam, which is described here as a cluster containing N particles with charges Z_i, can be cast in the form [6]

$$S_{\mathrm{cl}} = \langle dE/dx \rangle = \sum_i Z_i^2 S_p + \sum_{i \neq j} Z_i Z_j I(r_{ij}, v) , \qquad (1)$$

with

$$S_p = \frac{2}{\pi v^2} \int \frac{dk}{k} \int_0^{kv} \omega \, d\omega \, \mathrm{Im} \left[\frac{-1}{\varepsilon(k,\omega)} \right] \qquad (2)$$

and

$$I(r,v) = \frac{1}{2\pi^2 v} \int d^3k \frac{\mathbf{k}\cdot\mathbf{v}}{k^2} \mathrm{Im} \left[\frac{-1}{\varepsilon(k,\omega)} \right] \cos(\mathbf{k}\cdot\mathbf{r}) \qquad (3)$$

in terms of the beam velocity v and plasma dielectric function $\varepsilon(k,\omega)$, where $\omega = \mathbf{k}\cdot\mathbf{v}$; $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ denotes the distance between ions i and j. In this paper we will refer to the case of homonuclear clusters of atomic ions with equal effective charges $Z_i = Z_j = Z$ (the treatment of nonhomonuclear and molecular clusters has been considered in Refs. [8,9]).

In some previous studies the ion cluster was represented simply by the density $n_b(r)$, which provides an average of the actual particle distribution; this description has a shortcoming in that it neglects all the details of the correlation between the neighboring particles in the cluster. To overcome this problem we will use a description based on the pair-distribution function for the cluster, $g_{\mathrm{cl}}(r)$. In this way, a good statistical representation of these correlations can be achieved. Following the description made in Ref. [9] we introduce the function $g_{\mathrm{cl}}(r)$ for the cluster (or beam of particles) normalized according to

$$n_b \int d^3r g_{\mathrm{cl}}(r) = N - 1 , \qquad (4)$$

where n_b is the average density of the beam and N is the total number of particles (i.e., the number of particles in a given bunch).

We will consider here a spherical average of the interference terms and of the pair-distribution function (these approximations can be justified for systems with large numbers of interacting particles [9]). In particular, it has been shown that a very good average approximation for a random cluster of many ions ($N \gg 1$) can be obtained using the function

$$g_{cl}(r) = C_N \theta(r - r_0) p(r/2r_{cl}) . \quad (5)$$

Here r_{cl} is the radius of the cluster and $\theta(x)$ is the Heaviside function, which introduces an exclusion or *correlation volume* of effective radius r_0 (or the order of the interparticle distance d) around each beam particle [9], and is given by $(4\pi/3) r_0^3 n_b = 1$. The function $p(x)$ incorporates the effect of the finite cluster size into the probability of finding a pair of particles at distance r; it has the simple expression [9] $p(x) = [1 - (\frac{3}{2})x + (\frac{1}{2})x^3]\theta(1-x)$ [note that $p(x)$ vanishes if $x > 1$, i.e., $r > 2r_{cl}$]. Finally, C_N is a normalization constant, determined from Eq. (4), by

$$C_N = (N-2)/[N(1 - 8x_0^3 + 9x_0^4 - 2x_0^6)] ,$$

with

$$N = (4\pi/3) r_{cl}^3 n_b \text{ and } x_0 = r_0/2r_{cl} .$$

Therefore, we can write the mean energy loss (stopping power) for a beam of correlated ions as

$$S_{cl} = NZ^2(S_p + I_{cl}) = S_{ind} + \Delta S_{col} , \quad (6)$$

where $S_{ind} = NZ^2 S_p$ is the equivalent energy loss of independent ions (in terms of the proton stopping power S_p), and $\Delta S_{col} = NZ^2 I_{cl}$ is the collective effect in the energy loss, given by the average of the interference terms for the whole cluster:

$$I_{cl} = n_b \int d^3r\, g_{cl}(r) I(r,v) . \quad (7)$$

Stopping power calculations using the dielectric function approach both for classical and quantum mechanical plasmas have been described in detail in previous references [15–18]. In order to model the collective effects for a large cluster of ions, we first consider the calculation of interference effects for the simplest case of a pair of ions (a *dicluster*).

We show in Fig. 1 the values of the interference term $I(r,v)$ in Eqs. (1) and (3), as well as the stopping term $S_p(v)$ given by Eq. (2). The calculations correspond to a $r = 150$ a.u. (the equivalent beam density is $n_b = 4.8 \times 10^{17}$ cm^{-3}), plasma density $n_p = 3 \times 10^{22}$ cm^{-3}, and temperature $T = 300$ eV. In the following the values for distances, velocities, and plasma frequencies will be given in atomic units.

The points in Fig. 1(a) show the values obtained by the numerical integrations of Eqs. (2) and (3). The results, denoted by RPA (random-phase approximation), have been calculated according to Refs. [16,17]. The lines show high-velocity approximations derived here, namely

$$S_p \cong \left[\frac{e\omega_p}{v}\right]^2 \left[\ln\left[\frac{k_D}{k_{min}}\right] + F_2(v/v_T)\ln\left[\frac{k_{max}}{k_D}\right]\right] , \quad (8a)$$

$$I(r,v) \cong \left[\frac{e\omega_p}{v}\right]^2 [F_1(k_{min}r, k_D r) + F_2(v/v_T) F_1(k_D r, k_{max} r)] , \quad (8b)$$

with $F_1(x,y) = \text{Ci}(y) - \text{Ci}(x) + \sin(x)/x - \sin(y)/y$ [where $\text{Ci}(x)$ denotes the cosine integral] and $F_2(u) = (2/\pi)^{1/2} \int_0^u u^2 \exp(-u^2/2) du$; with $k_{min} = \omega_p/v$ and $k_D = \omega_p/v_T$, in terms of the plasma frequency ω_p and thermal velocity $v_T = \sqrt{k_B T/m}$. Clearly, in this range of velocities these approximations provide quite satisfactory results.

In Fig. 1(b) we have separated from the integrals of Eqs. (2) and (3) the contributions from collective ($k < k_D$) and individual ($k > k_D$) interactions to the interference effects. Values for $r = 150$ and similar calculations with

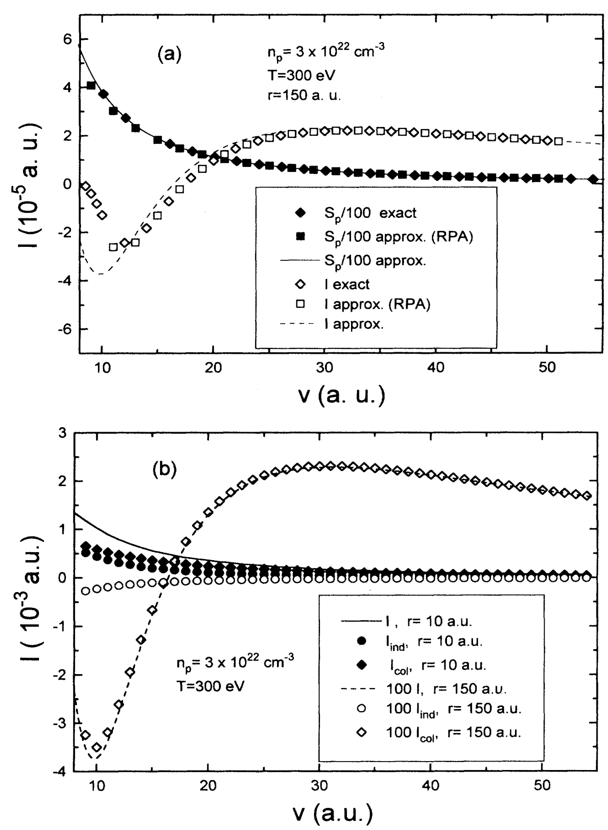

FIG. 1. Calculations of the interference function $I(r,v)$ from Eq. (3) and proton stopping term S_p from Eq. (2), versus beam velocity v. The calculations correspond to $r = 150$ a.u. (the equivalent beam density is $n_b = 4.8 \times 10^{17}$ cm^{-3}). The solid and dashed lines in part (a) show the analytical approximations of Eqs. (8a) and (8b); the stopping power values have been divided by 100. In part (b) we have separated the contributions from individual (circles) and collective (diamonds) terms, for the cases $r = 10$ and 150; here the dashed and solid lines show the total values of $I(r,v)$.

$r=10$ are shown. It can be seen that for the larger r values the collective terms dominate over the individual ones, while both terms become comparable for $r=10$ or smaller. Values of r of the order of 150 or larger are in the range of interest for high-intensity ion beams [3,13], where the collective terms become dominant.

Let us consider now the integration of the collective effect for a large cluster of ions according to Eqs. (5)–(7). We show in Fig. 2 the relevant functions: interference term $I(r\omega_p/v)$ (dashed line), cluster "shape" function $4\pi r^2 g_{\rm cl}(r)$ (dash-dot line), and the product of both (solid line) vs the distance r for the case $v=7$, $r_{\rm cl}=2.5\times10^4$, $n_p=10^{18}$ cm^{-3}, $k_BT=20$ eV (these values are in the range of interest for beam-plasma experiments in Z-pinch systems [19]). Therefore the integration in Eq. (7) will perform an average of positive and negative interferences between particles located at various distances within the cluster range. Since the lower limit in the integration is given by r_0 [cf. Eq. (5)], the integrated values will be sensitive to the distances r_0 between neighboring particles in the cluster, i.e., to the internal particle correlations (vicinage effect).

Integrations for large clusters are shown in Fig. 3 for plasma densities in the range 10^{18}–3×10^{22} cm^{-3}, and temperatures between 20 and 300 eV for clusters with $r_{\rm cl}=5.5\times10^5$ and various densities $n_b=(4\pi r_0^3/3)^{-1}$. By making a comparison with the proton stopping term S_p, we find that the magnitude of the collective effects turns out to be more important with increasing velocities. In fact, the collective terms may become completely dominant at the largest velocities, as in the case illustrated in Fig. 3(c). This behavior stems from Eqs. (4) and (7), which show that the upper limit to $I_{\rm cl}$ is $(N-1)S_p$ [in the extreme case where $I(r,v)\cong S_p$ for most of the ion pairs].

It is interesting to note also that the oscillations in Figs. 3(a) (where $n_p=3\times10^{22}$, $\omega_p=0.236$) and 3(c) ($n_p=10^{18}$, $\omega_p=1.36\times10^{-3}$) are of different origins. While the former are related to vicinage effects involving neighboring particles ($2\pi v/\omega_p\cong r_0$), the latter correspond to a coherent behavior of distant particles within the whole cluster range ($2\pi v/\omega_p\cong r_{\rm cl}$); this is in fact the *strong collective regime.* The sensitivity to the r_0 values mentioned before could be important for the details in the oscillatory behavior shown in Fig. 3(a), but not in the other cases.

A point of much interest is to show also the dependence of the interference effects on cluster size. Since the scale of distances for the interference effect is represented by $\lambda_{\max}=2\pi v/\omega_p$, one should expect that for a cluster radius much larger than $\lambda_{\max}$ the interference effect will approach a limiting value. Therefore, a *saturation* effect is

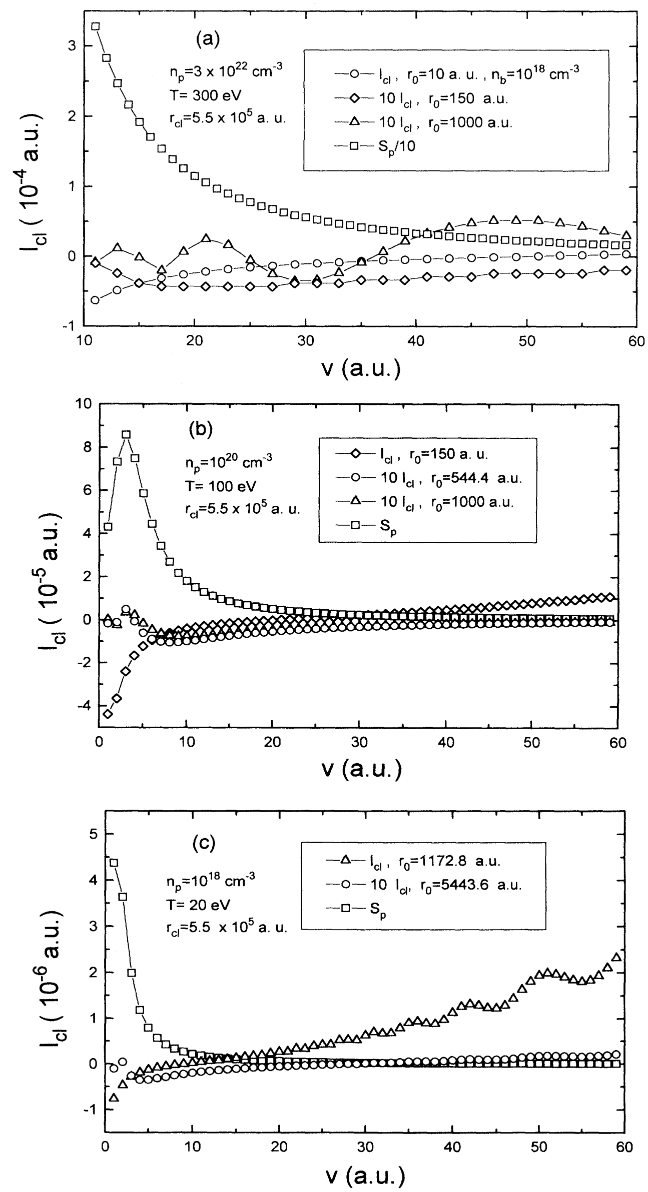

FIG. 3. Integrations of the cluster interference term $I_{\rm cl}$ for three plasma densities and temperatures, as indicated for each case. The proton stopping term S_p is also included for comparison.

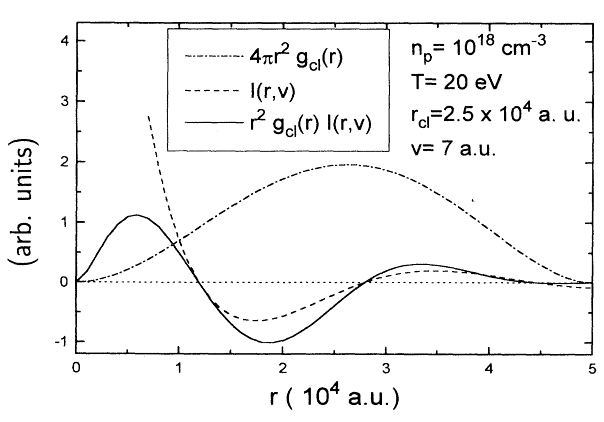

FIG. 2. Integration of the interference term $I_{\rm cl}$ in Eq. (7) for $n_p=10^{18}$ cm^{-3}, $k_BT=20$ eV, $r_{\rm cl}=2.5\times10^4$, and $v=7$. The dotted line shows the *envelope* function $4\pi r^2 g_{\rm cl}(r)$, the dashed line gives the function $I(r,v)$, and the solid line is the product of both [the integrand in Eq. (7)].

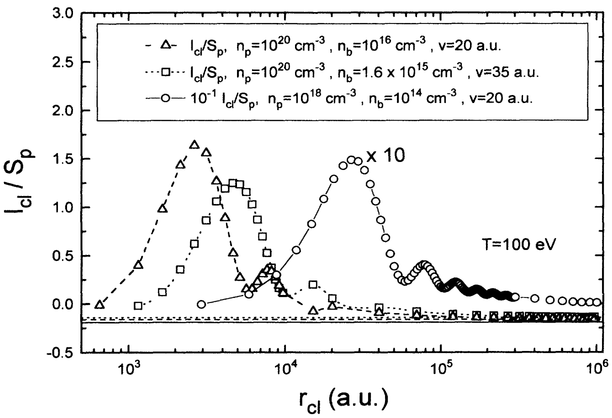

FIG. 4. Ratio between cluster and individual (proton) stopping power terms in Eq. (6) as a function of the cluster size $r_{\rm cl}$, for various beam and plasma conditions, cases (a), (b), and (c), as described in the text. The values of $I_{\rm cl}/S_p$ for case (c) have been divided by 10 (the maximum value is $\cong 15$).

expected.

This effect is shown in Fig. 4, where we show the ratio $I_{\rm cl}/S_p$ versus $r_{\rm cl}$ for a plasma with $k_B T = 100$ eV, and for the following beam parameters: (a) $n_p = 10^{20}$ cm^{-3}, $n_b = 10^{16}$ cm^{-3} ($r_0 = 544.4$), $v = 20$; (b) $n_p = 10^{20}$ cm^{-3}, $n_b = 1.6\times 10^{15}$ cm^{-3} ($r_0 = 1000$), $v = 35$; (c) $n_p = 10^{18}$ cm^{-3}, $n_b = 10^{14}$ cm^{-3} ($r_0 = 2526$), $v = 20$. Saturation values in the range of -10% to -20% are obtained for very large values of $r_{\rm cl}$.

However, the most striking feature here is the strong enhancement of the $I_{\rm cl}/S_p$ values for the intermediate range of $r_{\rm cl}$ (with maximum values $\cong 1.6$, 1.3, and 15 for $r_{\rm cl} \cong 3\times 10^3$, 5×10^3, and 3×10^4 in the cases shown in Fig. 4). To find an explanation for this effect one has to consider in some detail the behavior of the cluster interference functions shown in Fig. 2. From the analysis of these functions we found that the maximum of $I_{\rm cl}/S_p$ occurs for $r_{\rm cl} \cong 2v/\omega_p$ and that the values of this maximum can be approximated by

$$[I_{\rm cl}/S_p]_{\max} \cong \tfrac{1}{3} n_b (v/\omega_p)^3 \ . \qquad (9)$$

At this value of $r_{\rm cl}$ one finds a positive interference for most of the particles in the cluster (corresponding to an $r_{\rm cl}$ value close to the first maximum in the solid line curve of Fig. 2). This is the case of *maximum coherent behavior,* which produces a prominent enhancement in the $I_{\rm cl}/S_p$ values. With increasing cluster size, the positive and negative interferences in Fig. 2 are averaged in such a way that the final saturation values of $I_{\rm cl}/S_p$ for very large clusters become much smaller than the maximum value (and could also have a different sign). In the examples shown in Fig. 4, the number of particles $N = (4\pi/3) n_b r_{\rm cl}^3$ corresponding to the maximum values of $I_{\rm cl}/S_p$ are 135, 156, and 1560.

TABLE I. Illustrative values of ion-beam parameters corresponding to a maximum collective effect of 100% for the case of Tokamak, Z-pinch, and ICF plasmas, for beam velocities $v = 10$, 20, and 40 a.u. The number of particles in the cluster in all cases is $N \approx 100$.

Plasma	n_P (cm^{-3})	v (a.u.)	n_b (cm^{-3})	$r_{\rm cl}$ (a.u.)
Tokamak	10^{14}	10	5×10^7	7.7×10^5
Z-Pinch	10^{18}	20	6.4×10^{12}	1.5×10^4
ICF	10^{22}	40	7.9×10^{17}	3.1×10^2

Thus, a remarkable consequence of this behavior is that the maximum enhancement in the energy losses due to the collective effect is obtained not for the largest clusters, but for those whose dimensions are of the order of $2v/\omega_p$. This value corresponds to the conditions for maximum overall interference between particles in the cluster.

Therefore, in order to maximize the energy deposition in the plasma one should consider clusters or particle bunches with the appropriate parameters corresponding to

$$r_{\rm cl} \cong 2v/\omega_p \ , \quad N \cong (4\pi/3) n_b (2v/\omega_p)^3 \ , \qquad (10)$$

Some illustrative examples for Tokamak, Z-pinch, and ICF plasmas are contained in Table I, where we show the values of the beam parameters required to obtain a 100% increase in the energy loss, so that

$$\tfrac{1}{3} n_b (v/\omega_p)^3 \cong 1 \ .$$

The values in this table correspond to ion velocities $v = 10$, 20, and 40 a.u. and typical plasma densities.

In summary, we have formulated a model to describe the collective effects in the energy loss of intense ion beams or large ion clusters in plasmas. Important interference effects are obtained that depend on the dynamical vicinage interactions between the particles within a given bunch or cluster. The magnitude of the collective effects in the energy loss increases with beam velocity, and the dependence on cluster size shows a strong enhancement for sizes comparable to the dynamical range of interactions between swift ions in the plasma.

The present description would be useful in estimating the magnitude of the collective effects in ICF, or in magnetic confinement experiments using high-intensity particle beams. Further calculations and applications to other cases of interactions between more compact ion clusters and dense plasmas will be given elsewhere.

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